In the case of mass and heat addition in a supersonic flow past a curved surface given by the equation $y = h(x)$, we obtain the following relations for the velocity perturbations at the wall:

$$w'_x = -\frac{w_\infty}{\sqrt{M_\infty^2 - 1}}\frac{d}{dx}h(x) - \frac{k-1}{Aa_\infty^2}\frac{1}{(M_\infty^2-1)}\times$$

$$\times\int_{-\infty}^{x}\dot{q}\left[\mu,\ h(x)+\frac{x-\mu}{\sqrt{M_\infty^2-1}}\right]d\mu - \frac{1}{(M_\infty^2-1)}\times\int_{-\infty}^{x}\dot{m}\left[\mu,\ h(x)+\frac{x-\mu}{\sqrt{M_\infty^2-1}}\right]d\mu,$$

$$w'_y = w_\infty\frac{d}{dx}h(x) + \frac{k-1}{Aa_\infty^2}\frac{1}{\sqrt{M_\infty^2-1}}\times$$

$$\times\int_{-\infty}^{x}\dot{q}\left[\mu,\ h(x)-\frac{x-\mu}{\sqrt{M_\infty^2-1}}\right]d\mu + \frac{1}{\sqrt{M_\infty^2-1}}\times$$

$$\times\int_{-\infty}^{x}\dot{m}\left[\mu,\ h(x)-\frac{x-\mu}{\sqrt{M_\infty^2-1}}\right]d\mu.$$

Expressions for the velocity perturbations in flow over a surface may be obtained by the analogous method given in [1].

NOTATION

p – pressure; ρ – density; T – temperature; R – gas constant; k – adiabatic exponent; a – speed of sound; c_p – specific heat at constant pressure; w_x, w_y, w_z – velocity components along the rectangular axes x, y, z; $\dot{q}$, $\dot{m}$ – amount of heat and mass, respectively, supplied to the flow in unit time referred to unit mass of gas; Δq and Δm – amount of heat and mass, respectively, supplied to unit mass of gas of the volume in question; i_{0T} – enthalpy of added mass; A – mechanical equivalent of heat; t – time; Φ – velocity potential. Subscripts: ∞ – free-stream parameters; prime – perturbations of corresponding parameter.

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20 August 1964

The Dnepropetrovsk 300-Year Reunification of the Ukraine with Russia State University

UDC 536.41

TEMPERATURE STRESSES IN A LONG PRISM OF RECTANGULAR CROSS SECTION

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Inzhenerno-Fizicheskii Zhurnal, Vol. 8, No. 5, pp. 687-691, 1965

Following publication of my article [1] on determination of the temperature stresses in a long prism of rectangular cross section, R. S. Minasyan (Erevan) pointed out that conditions (7)-(10)* are not satisfied on the interval (0, 1), except possibly for a finite number of points.

To obtain the conditions which the functions $\varphi_k(\pm 1)$ and $\psi_k(\pm 1)$, must satisfy, it is necessary to integrate (from 0 to 1) the system of equations preceding (7)-(10), namely,

*The numbering is that of [1].

$$\sum_{k=1}^{\infty}[-\nu_k^2\cos\nu_k Y\,\varphi_k(\pm 1)+\psi_k''(Y)\cos\mu_k]=$$

$$=-\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_nB_m\frac{\nu_m^2\cos\mu_n\cos\nu_m Y}{\mu_n^2+\nu_m^2}, \tag{1}$$

$$\sum_{k=1}^{\infty}[-\mu_k^2\psi_k(\pm 1)\cos\mu_k X+\varphi_k''(X)\cos\nu_k]=$$

$$=-\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_nB_m\frac{\mu_n^2\cos\mu_n X\cos\nu_m}{\mu_n^2+\nu_m^2}, \tag{2}$$

$$-\sum_{k=1}^{\infty}[\varphi_k'(\pm 1)\nu_k\sin\nu_k Y+\psi_k'(Y)\mu_k\sin\mu_k(\pm 1)]=$$

$$=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_nB_m\frac{\mu_n\nu_m\sin\mu_n(\pm 1)\sin\nu_m Y}{\mu_n^2+\nu_m^2}, \tag{3}$$

$$-\sum_{k=1}^{\infty}[\varphi_k'(X)\nu_k\sin\nu_k(\pm 1)+\psi_k'(\pm 1)\mu_k\sin\mu_k X]=$$

$$=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_nB_m\mu_n\nu_m\frac{\sin\mu_n X\sin\nu_m(\pm 1)}{\mu_n^2+\nu_m^2}, \tag{4}$$

after first multiplying (1) by $\cos\nu_k Y$, (2) by $\cos\mu_k X$, (3) by $\sin\nu_k Y$, and (4) $\sin\mu_k X$.

As a result of integration we get

$$-a_k\nu_k^2\varphi_k(\pm 1)+\cos\mu_k\int_0^1\psi_k''(Y)\cos\nu_k Y\,dY=-a_k\nu_k^2B_k\sum_{n=1}^{\infty}\frac{A_n\cos\mu_n}{\mu_n^2+\nu_k^2}, \tag{5}$$

$$-b_k\mu_k^2\psi_k(\pm 1)+\cos\nu_k\int_0^1\varphi_k''(X)\cos\mu_k X\,dX=$$

$$=-b_k\mu_k^2A_k\sum_{m=1}^{\infty}\frac{B_m\cos\nu_m}{\mu_k^2+\nu_m^2}, \tag{6}$$

$$-a_k'\nu_k\varphi_k'(\pm 1)+\mu_k\sin\mu_k(\pm 1)\int_0^1\psi_k'(Y)\sin\nu_k Y\,dY=$$

$$=a_k'\nu_kB_k\sum_{n=1}^{\infty}\frac{\mu_nA_n\sin\mu_n(\pm 1)}{\mu_n^2+\nu_k^2}, \tag{7}$$

$$b_k'\mu_k\psi_k'(\pm 1)-\nu_k\sin\nu_k(\pm 1)\int_0^1\varphi_k'(X)\sin\mu_k X\,dX=$$

$$=b_k'\mu_kA_k\sum_{m=1}^{\infty}\frac{\nu_mB_m\sin\nu_m(\pm 1)}{\mu_k^2+\nu_m^2}, \tag{8}$$

where

$$a_k = \frac{1}{2} + \frac{\sin 2\nu_k}{4\nu_k}, \quad a_k' = \frac{1}{2} - \frac{\sin 2\nu_k}{4\nu_k},$$

$$b_k = \frac{1}{2} + \frac{\sin 2\mu_k}{4\mu_k}, \quad b_k' = \frac{1}{2} - \frac{\sin 2\mu_k}{4\mu_k}.$$

Using the expressions adopted in [1] for the functions $\varphi_k = (x)$ and $\psi_k(Y)$, we obtain from (4)-(8) the following set of equations for determining the coefficients C_k, D_k, F_k, etc.:

$$C_k \operatorname{ch}\nu_k + D_k \operatorname{sh}\nu_k + E_k \operatorname{ch}\nu_k + F_k \operatorname{sh}\nu_k +$$

$$+ C_k'\left(\frac{\mu_k^2 \cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{ch}\mu_k Y dY\right) + D_k'\left(\frac{\mu_k^2\cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{sh}\mu_k Y dY\right) +$$

$$+ E_k'\left(\frac{2\mu_k\cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{sh}\mu_k Y dY + \frac{\mu_k^2\cos\mu_k}{\nu_k^2 a_k}\int_0^1 Y\cos\nu_k Y \operatorname{ch}\mu_k Y dY\right) +$$

$$+ F_k'\left(\frac{2\mu_k\cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{ch}\mu_k Y dY + \frac{\mu_k^2\cos\mu_k}{\nu_k^2 a_k}\int_0^1 Y\cos\nu_k Y \operatorname{sh}\mu_k Y dY\right) =$$

$$= -B_k \sum_{n=1}^{\infty} \frac{A_n \cos\mu_n}{\mu_n^2 + \nu_k^2}; \tag{9}$$

$$C_k \operatorname{ch}\nu_k - D_k \operatorname{sh}\nu_k - E_k \operatorname{ch}\nu_k + F_k \operatorname{sh}\nu_k +$$

$$+ C_k'\left(\frac{\mu_k^2 \cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{ch}\mu_k Y dY\right) + D_k'\left(\frac{\mu_k^2\cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{sh}\mu_k Y dY\right) +$$

$$+ E_k'\left(\frac{2\mu_k\cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{sh}\mu_k Y dY + \frac{\mu_k^2\cos\mu_k}{\nu_k^2 a_k}\int_0^1 Y\cos\nu_k Y \operatorname{ch}\mu_k Y dY\right) +$$

$$+ F_k'\left(\frac{2\mu_k\cos\mu_k}{\nu_k^2 a_k}\int_0^1 \cos\nu_k Y \operatorname{ch}\mu_k Y dY + \frac{\mu_k^2\cos\mu_k}{\nu_k^2 a_k}\int_0^1 Y\cos\nu_k Y \operatorname{sh}\mu_k Y dY\right) =$$

$$= -B_k \sum_{n=1}^{\infty} \frac{A_n \cos\mu_n}{\mu_n^2 + \nu_k^2}; \tag{10}$$

$$C_k\left(\frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1 \cos\mu_k X \operatorname{ch}\nu_k X dX\right) + D_k\left(\frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1 \cos\mu_k X \operatorname{sh}\nu_k X dX\right) +$$

$$+ E_k\left(\frac{2\nu_k\cos\nu_k}{\mu_k^2 b_k}\int_0^1 \cos\mu_k X \operatorname{sh}\nu_k X dX + \frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1 X\cos\mu_k X \operatorname{ch}\nu_k X dX\right) +$$

$$+ F_k\left(\frac{2\nu_k\cos\nu_k}{\mu_k^2 b_k}\int_0^1 \cos\mu_k X \operatorname{ch}\nu_k X dX + \frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1 X\cos\mu_k X \operatorname{sh}\nu_k X dX\right) -$$

$$- C_k' \operatorname{ch}\mu_k - D_k' \operatorname{sh}\mu_k - E_k' \operatorname{ch}\mu_k - F_k' \operatorname{sh}\mu_k = -A_k \sum_{m=1}^{\infty} \frac{B_m \cos\nu_m}{\mu_k^2 + \nu_m^2}; \tag{11}$$

$$C_k\left(\frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1\cos\mu_k X \operatorname{ch}\nu_k X dX\right)+D_k\left(\frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1\cos\mu_k X\operatorname{sh}\nu_k XdX\right)+$$

$$+E_k\left(\frac{2\nu_k\cos\nu_k}{\mu_k^2 b_k}\int_0^1\cos\mu_k X\operatorname{sh}\nu_k XdX+\frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1 X\cos\mu_k X\operatorname{ch}\nu_k XdX\right)+$$

$$+F_k\left(\frac{2\nu_k\cos\nu_k}{\mu_k^2 b_k}\int_0^1\cos\mu_k X\operatorname{ch}\nu_k XdX+\frac{\nu_k^2\cos\nu_k}{\mu_k^2 b_k}\int_0^1 X\cos\mu_k X\operatorname{sh}\nu_k XdX\right)-$$

$$-C_k'\operatorname{ch}\mu_k+D_k'\operatorname{sh}\mu_k+E_k'\operatorname{ch}\mu_k-F_k'\operatorname{sh}\mu_k=-A_k\sum_{m=1}^{\infty}\frac{B_m\cos\nu_m}{\mu_k^2+\nu_m^2}; \qquad (12)$$

$$-C_k\nu_k\operatorname{sh}\nu_k-D_k\nu_k\operatorname{ch}\nu_k-E_k(\nu_k\operatorname{sh}\nu_k+\operatorname{ch}\nu_k)-F_k(\nu_k\operatorname{ch}\nu_k+\operatorname{sh}\nu_k)+$$

$$+C_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{sh}\mu_k YdY\right)+D_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{ch}\mu_k YdY\right)+$$

$$+E_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1 Y\sin\nu_k Y\operatorname{sh}\mu_k YdY+\frac{\mu_k\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{ch}\mu_k YdY\right)+$$

$$+F_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1 Y\sin\nu_k Y\operatorname{ch}\mu_k YdY+\frac{\mu_k\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{sh}\mu_k YdY\right)=$$

$$=B_k\sum_{n=1}^{\infty}\frac{\mu_n A_n\sin\mu_n}{\mu_n^2+\nu_k^2}; \qquad (13)$$

$$C_k\nu_k\operatorname{sh}\nu_k-D_k\nu_k\operatorname{ch}\nu_k-E_k(\nu_k\operatorname{sh}\nu_k+\operatorname{ch}\nu_k)+F_k(\nu_k\operatorname{ch}\nu_k+\operatorname{sh}\nu_k)-$$

$$-C_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{sh}\mu_k YdY\right)-D_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{ch}\mu_k YdY\right)-$$

$$-E_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1 Y\sin\nu_k Y\operatorname{sh}\mu_k YdY+\frac{\mu_k\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{ch}\mu_k YdY\right)-$$

$$-F_k'\left(\frac{\mu_k^2\sin\mu_k}{\nu_k a_k'}\int_0^1 Y\sin\nu_k Y\operatorname{ch}\mu_k YdY+\frac{\mu_k\sin\mu_k}{\nu_k a_k'}\int_0^1\sin\nu_k Y\operatorname{sh}\mu_k YdY\right)=$$

$$=-B_k\sum_{n=1}^{\infty}\frac{\mu_n A_n\sin\mu_n}{\mu_n^2+\nu_k^2}; \qquad (14)$$

$$C_k\left(\frac{\nu_k^2\sin\nu_k}{\mu_k b_k'}\int_0^1\sin\mu_k X\operatorname{sh}\nu_k XdX\right)+D_k\left(\frac{\nu_k^2\sin\nu_k}{\mu_k b_k'}\int_0^1\sin\mu_k X\operatorname{ch}\nu_k XdX\right)+$$

$$+E_k\left(\frac{\nu_k^2\sin\nu_k}{\mu_k b_k'}\int_0^1 X\sin\mu_k X\operatorname{sh}\nu_k XdX+\frac{\nu_k\sin\nu_k}{\mu_k b_k'}\int_0^1\sin\mu_k X\operatorname{ch}\nu_k XdX\right)+$$

$$+F_k\left(\frac{\nu_k^2\sin\nu_k}{\mu_k b_k'}\int_0^1 X\sin\mu_k X\operatorname{ch}\nu_k XdX+\frac{\nu_k\sin\nu_k}{\mu_k b_k'}\int_0^1\sin\mu_k X\operatorname{sh}\nu_k XdX\right)-$$

$$-C_k' \mu_k \operatorname{sh} \mu_k - D_k' \mu_k \operatorname{ch} \mu_k - E_k' (\mu_k \operatorname{sh} \mu_k + \operatorname{ch} \mu_k) - F_k' (\mu_k \operatorname{ch} \mu_k + \operatorname{sh} \mu_k) =$$

$$= -A_k \sum_{m=1}^{\infty} \frac{\nu_m B_m \sin \nu_m}{\mu_k^2 + \nu_m^2}; \tag{15}$$

$$C_k \left(\frac{\nu_k^2 \sin \nu_k}{\mu_k b_k'} \int_0^1 \sin \mu_k X \operatorname{sh} \nu_k X dX \right) + D_k \left(\frac{\nu_k^2 \sin \nu_k}{\mu_k b_k'} \int_0^1 \sin \mu_k X \operatorname{ch} \nu_k X dX \right) +$$

$$+ E_k \left(\frac{\nu_k^2 \sin \nu_k}{\mu_k b_k'} \int_0^1 X \sin \mu_k X \operatorname{sh} \nu_k X dX + \frac{\nu_k \sin \nu_k}{\mu_k b_k'} \int_0^1 \sin \mu_k X \operatorname{ch} \nu_k X dX \right) +$$

$$+ F_k \left(\frac{\nu_k^2 \sin \nu_k}{\mu_k b_k'} \int_0^1 X \sin \mu_k X \operatorname{ch} \nu_k X dX + \frac{\nu_k \sin \nu_k}{\mu_k b_k'} \int_0^1 \sin \mu_k X \operatorname{sh} \nu_k X dX \right) -$$

$$- C_k' \mu_k \operatorname{sh} \mu_k + D_k' \mu_k \operatorname{ch} \mu_k + E_k' (\mu_k \operatorname{sh} \mu_k + \operatorname{ch} \mu_k) -$$

$$- F_k' (\mu_k \operatorname{ch} \mu_k + \operatorname{sh} \mu_k) = -A_k \sum_{m=1}^{\infty} \frac{\nu_m B_m \sin \nu_m}{\mu_k^2 + \nu_m^2}. \tag{16}$$

Thus, the temperature stresses in a long prism of rectangular cross section are determined, as before, by relations (2)-(6) of [1], but the coefficients C_k, D_k, E_k, etc. in the expressions for functions $\varphi_k(X)$ and $\psi_k(Y)$ must be found by solving the system (9)-(16), and not the system (11)-(18) of [1].

We note that the values of coefficients A_k and B_k decrease continuously as the Fourier numbers (Fo_1 and Fo_2) increase.

In the majority of cases of practical interest, all the coefficients A_k and B_k beyond the first one to three may be neglected in view of their smallness, in connection with which the system of equations (9)-(16) is of bounded order.

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23 June 1964

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